In the remaining chapters, attention shifts to surfaces. Rational Bézier patches play a central role. The bilinear and the bicubic cases are singled out. Surfaces of revolution and developable surfaces are considered specially. Triangular patches, quadric surfaces, and Gregory patches are topics considered in the later chapters. The fifteenth and last chapter gives some examples and a discussion of the IGES standards for NURBS. There is a good bibliography and a good index. Copious references to the literature are made throughout the book.

All-in-all, this is a very appealing book that should have a stimulating effect on the teaching of this important subject. It can certainly be recommended for solo study because of the gentle expository style of the writing.

E. W. C.

26[65-06, 65D05, 65D07, 65D17]—*Designing fair curves and surfaces*, Nickolas S. Sapidis (Editor), Geometric Design Publications, SIAM, Philadelphia, PA, 1994, xii+318 pp., $25\frac{1}{2}$ cm, softcover, $61.50

This volume, the seventh in a series of geometric design publications from SIAM, focuses on the problem of "visually appealing" line/surface construction. Its twelve chapters explore various ways of (i) defining "fairness" or "shape quality" mathematically, (ii) developing new curve and surface schemes that guarantee fairness, (iii) enabling a user to identify and remove local shape aberrations without global disturbance.

A common thread is the use of differential geometry constructs to express fairness. Thus, we encounter *arc length* s, *curvature* κ, *radius of curvature* $\rho = 1/\kappa$, and *torsion* τ in the study of lines, *principal curvatures* k_1, k_2, *mean curvature* $H = (k_1 + k_2)/2$, and *Gaussian curvature* $K = k_1 k_2$ in the study of surfaces. Typically, not these quantities alone, but also their arcwise derivatives (or divided differences), are the determinants of shape quality. In the volume's broadest-gauged chapter, Roulier and Rando list eight different fairness metrics for lines, of which the first, $\mu = \int [\rho^2 \tau^2 + (\rho')^2]^{1/2}\, ds$, is representative. They propose the minimization of μ over a preselected family of design curves as an answer to (i) and (ii) above. Surfaces are to be treated similarly, with at least five double-integral fairness metrics to choose from.

Other chapters present comparable schemes. Moreton and Séquin construct interpolatory quintic spline curves that minimize the functional $\int \|\vec{\kappa}'\|^2\, ds$, and biquintic surface patches wherein (loosely speaking) the total of such functionals over all lines of principal curvature is minimal. Eck and Jaspert work with point sets only. They interpolate data by a polygon, invoke *difference geometry* to obtain discrete curvature and torsion derivatives $\kappa_i, \kappa_i', \kappa_i'', \tau_i, \tau_i'$ at each inner vertex, and perturb these vertices iteratively, so as to minimize $\sum_i [(\kappa_i'')^2 + (\tau_i')^2]$. Feldman obtains discrete curvature in the same way for a planar polygon with vertices (L_i, X_i), and takes the length μ of the derived polygon (L_i, κ_i) as a fairness metric. His aim is to minimize μ by perturbing the ordinates X_i between prescribed tolerance limits.

Several authors prefer inequality constraints on $\kappa, \kappa', \ldots$ to the metric approach. Burchard et al. fit discrete points in the plane by a circular spline with curvature of uniform sign, monotone and log-convex as a function of s, between designated nodes. Ginnis et al. fit the same points by a polynomial spline of nonuniform degree. They allow small perturbation of the data, one point at a time, and local degree elevation as needed to satisfy certain shape-preservation and fairness criteria, holding κ and κ' to the fewest possible changes of sign. A. K. Jones proposes to fit planar data by a (parametric) polynomial spline whose curvature profile approximates, as closely as possible in a least squares sense, a user-supplied target profile and may also have to satisfy positivity and/or monotonicity and/or convexity constraints. Skillful handling leads to an optimization problem wherein both objective and constraint functions are polynomial in the unknown spline coefficients.

Rounding out the volume are various papers that speak to the issue of shape control without explicitly defining, or attempting to measure, fairness: Gallagher and Piper on convexity-preserving surface interpolation, Bloor and Wilson on interactive design using PDEs, J. Peters on surfaces of arbitrary topology using biquadratic and bicubic splines, Zhao and Rockwood on a convolution approach to N-sided patches, Beier and Chen on a simplified reflection model for interactive smoothness evaluation.

Weston Meyer
Mathematics Department
General Motors R & D Center
Warren, MI 48090-9055

27[12Y05, 65T10, 94B05]—*Computational number theory and digital signal processing: Fast algorithms and error control techniques*, by Hari Krishna, Bal Krishna, Kuo-Yu Lin and Jenn-Dong Sun, CRC Press, Boca Raton, FL, 1994, xviii+330 pp., 24 cm, $59.95

Two main topics of this book are number-theoretic transforms and fast algorithms for the calculation of convolutions in digital signal processing. The relevant parts of the book are written in the same spirit as the classical monograph of McClellan and Rader [2]. The underlying algebraic structures are the residue class rings $Z(M)$ of the integers modulo M as well as polynomial rings over $Z(M)$. The third main topic of the book is error detection and correction by linear codes over $Z(M)$, with special reference to fault tolerance in modular arithmetic. It will be hard to find the material on this topic in any other book.

The authors appear to be comfortable with the algorithmic and signal processing aspects of their material, but the treatment of the algebraic background leaves a lot to be desired. The Chinese Remainder Theorem is proved several times over for various rings, when it would have been more efficient to establish once and for all the general Chinese Remainder Theorem for rings as in Lang [1, Chapter II]. Several basic definitions are wrong. For instance, on p. 32 it is said that two polynomials are relatively prime if they have no factors in common, and a similar error occurs on p. 34 in the definition of irreducible polynomials. In the definition of the order of an element on p. 46, replace "smallest non-zero value" by "smallest positive value". The book is replete with awkward formulations such as "A polynomial $A(u)$ is called *monic* if the coefficient of its highest degree is equal to 1" (Definition 3.2) and "Here the entire theorem is defined over the ring $Z(p^\alpha)$" (Theorem 4.6). On p. 80 we read that "It is necessary and sufficient that $P(u)$ be monic and of degree greater than $n-1$", but it is not said for which property this is necessary and sufficient. On p. 55 replace "Legrange" by "Lagrange". This is just a sample selection of deficiencies.